Quality Factor Comparison of Coaxial-Fed and Edge-Fed Electrically Small Microstrip Antennas

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***Abstract*—This letter presents and discusses the simulated and measured values of Q for microstrip antennas with different feeding schemes (edge-fed versus coaxial-fed) for regular (rectangular) and electrically small (meandered) designs. The edge-fed antennas are not matched, while the coaxial-fed are matched to 50 Ω. The measured Q is compared to that given by exact and approximate formulas, simulations, and the Chu lower bound. The objective is to assess by how much a measurement of Q typically differs from its anticipated value, in regular and miniaturized microstrip antennas for different feeding schemes.**

***Index Terms*—Chu limit, electrically small antennas (ESAs), microstrip antennas, quality factor Q.**

I. INTRODUCTION

ELECTRICALLY small antennas (ESAs) have, in general, small input resistance R_{in} that is not always trivial to match. Microstrip antennas have been miniaturized and made electrically small by etching slits parallel to their radiating edges to increase the effective length while maintaining the overall antenna size [1], [2]. The effectiveness of such miniaturization techniques can be assessed using the antenna quality factor Q. Wheeler [3], [4] and Chu [5] showed that the antenna radiation Q relates to the wavelength λ and the antenna physical size. Exact and approximate formulas for Q have been developed [6]–[8]. These formulas have been proven theoretically and validated for wire-based structures such as dipoles and spherical helices [9], [10]. New results for the minimum values of Q were given in [11] and [12], while recently [13] an assessment for the popular microstrip antennas was made. Although mathematically the antenna Q can be independent of the feed and matching scheme [6], in practice the measured Q may differ.

Approximate expressions used for the calculation of the antenna quality factor Q do not calculate Q at resonance. Thus, they do not take into account if the antenna is matched or not, which can be directly linked to the antenna's type of feed. As a result, Q and consequently the associated bandwidth are usually determined by the size of the antenna element [14] and ought to be independent of the type of feed. In practice, however, the measured Q may differ from the anticipated due to even slight measurement or fabrication inaccuracies and the inductance of the feed probe implemented in the coaxial feed. The objective of this letter is to assess by how much the measured Q typically differs from the anticipated for regular and miniaturized microstrip antennas of different feeding schemes. To achieve that, we compare the Q of microstrip antennas with different feeds (edge-fed versus coaxial-fed) for regular (rectangular) and electrically small (meandered) microstrip antennas that we developed. The edge-fed antennas are mismatched, while the coaxial-fed are matched to 50 Ω. The measured Q is compared to the Q obtained by exact, approximate formulas, simulations, and the Chu lower bound. Comparison shows the anticipated similarities, but also reveals differences and characteristics, while it provides useful insights on antenna characterization.

II. THEORETICAL BACKGROUND AND Q DEFINITIONS

The Q of an antenna is an important overall parameter specifying the antenna performance and the inherent physical limitations of antenna size on the gain. High Q means a large amount of reactive energy is stored in the near-zone field. This in turn implies large currents, high ohmic losses, narrow bandwidth, and large frequency sensitivity. The knowledge of antenna Q leads to a reasonably definite assessment of the antenna performance because of its clear physical implication. As a measure of antenna performance, the quality factor Q can be defined generally in three ways: 1) as a relation between the stored reactive energy and radiated power of the antenna; 2), as a function of its impedance or admittance; and 3) as a function of its bandwidth. Every approach has its own physical meaning and yields different operational limits.

The first approach gives an *exact* quality factor in terms of the angular resonant frequency ω_c, the time-average stored magnetic and electric energy in the antenna, and the total power dissipated in radiation and losses. These quantities are derived from the antenna impedance and fields, the latter however can be difficult to measure

$$Q_0(\omega_0)=\left|\frac{\omega_0}{2R_0(\omega_0)}X_0'(\omega_0)-\frac{2\omega_0}{|I_0|^2R_0(\omega_0)}\left[W_L(\omega_0)+W_R(\omega_0)\right]\right| \tag{1}$$

with $W_L(\omega_0) \approx \mathrm{Im}\int_{4\pi} R(\ell)I'(\ell)I^*(l)d\ell$ and $W_R(\omega_0) = (1/2Z_{\mathrm{f}})\int_{4\pi}\mathrm{Im}(F'F^*)d\Omega$ the material-loss and far field dispersion energy, respectively, which depend on the frequency derivative of the fields [6].

While the exact expression of Q defines an upper bound value, a restriction is also imposed concerning its lower value. When an antenna becomes electrically small, its bandwidth decreases. The Q is related to the radiansphere, the space the

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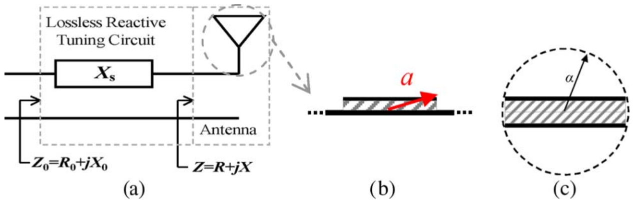


Fig. 1. (a) Schematic of a general transmitting antenna and the additional series, lossless reactive element. (b) Theoretical model of a microstrip antenna on finite dielectric over infinite ground. (c) Approximate microstrip antenna model and its radiansphere of radius a, applicable to microstrip antennas using image theory for infinite ground. Model is exact for air dielectric substrate.

antenna fills, and whose diameter was related by Chu to the fundamental Q limitation, which McLean later reexamined [15]

$$Q_{lb} = \frac{1}{k^3a^3} + \frac{1}{ka} \tag{2}$$

where $k = 2\pi/\lambda$, λ the wavelength, and α the radiansphere radius.

Expression (2) holds for linearly polarized antennas and establishes a fundamental limit no antenna can exceed. The more efficiently an antenna occupies its radiansphere, the closer its Q would be to this Chu lower bound, and the larger bandwidth it will have.

Using the second approach, an *approximate* expression of Q can be derived that is based on the antenna input resistance and reactance [6]. This approach is of significant interest in this letter because it provides an accurate value for Q directly from the antenna complex $Z_{\rm in}$ that can easily be measured

$$Q_{\rm z}(\omega) \approx \frac{\omega}{2R(\omega)}\sqrt{R'(\omega)^2 + \left(X'(\omega) + \frac{|X(\omega)|}{\omega}\right)^2} \tag{3}$$

where $R(\omega)$, $X(\omega)$, and X_0' are the resistance and reactance of the antenna, without and with the series reactive element, respectively, shown in Fig. 1(a). Using (3), the antenna Q is found independently of the antenna feeding technique, which means that a matched and a mismatched antenna are expected to have the same Q value. Equation (3) is advantageous over the exact (1) because it does not require knowing the fields.

The third approach relates Q with the matched VSWR bandwidth, which is the difference of two frequencies adjacent to the resonant where VSWR has a constant value s. To find this Q, the characteristic impedance Z_0 of the antenna feedline is designed equal to the input resistance at resonance

$$Q_{{\rm FBW}_V}(\omega_0) \approx \frac{2\sqrt{\frac{s-1}{2\sqrt{2}}}}{\frac{\omega_+ - \omega_-}{\omega_0}}. \tag{4}$$

III. EVALUATION OF ANTENNA Q FOR DIFFERENT FEEDS

To evaluate the antenna Q for different feeds, the input impedance of various antennas was simulated and measured. From this $Z_{\rm in}$, the antenna Q was calculated using (1), (3), and (4). First, a *rectangular* microstrip antenna of resonant length $L = 36$ mm and width $W = 44$ mm was designed on $h = 0.75$ mm thick RO3003 with $\varepsilon_{\rm r} = 3$ and $\tan\delta = 0.0013$. With this antenna as a foundation, miniaturized *meander* microstrip antennas with the same dimensions were designed.

To feed these antennas, two methods were tested: 1) an edge feed (with a very short transmission line), and 2) a coaxial cable directly connected beneath the patch. Thus, four antenna prototypes in total (two rectangular, two meander) were fabricated first and are shown in Fig. 2. The edge-fed ones help examine the effect of mismatch on Q and evaluate if in practice it is indeed similar to that of coaxial-fed antennas, especially near the resonance, as explained theoretically in [6].

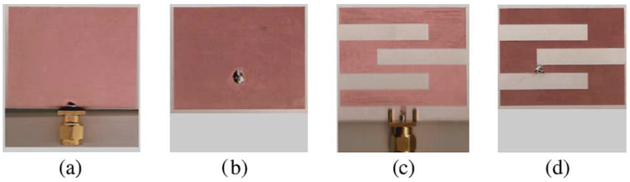


Fig. 2. Fabricated microstrip antennas (all 44 × 36 mm^2): (a) rectangular edge-fed, (b) rectangular coaxial-fed, (c) slitted edge-fed, and (d) slitted coaxial-fed.

TABLE I
EDGE- AND COAX-FED MICROSTRIP ANTENNA Q: SUBSTRATE THICKNESS, SLIT LENGTH, RESONANT FREQ., MEASURED AND SIMULATED APPROX. $Q_{\rm z}$ (2), SIM. MATCHED VSWR $Q_{\rm FBW}$ (4), SIM. EXACT Q_0 (1), $\Delta Q_{\rm z}$ (%) AND ka VALUE

Feed Type	**Thickness *h* (mm)**	**L_s (mm)**	**f_0 (GHz)**	**Q_z (Meas)**	**Q_z (Sim)**	**$Q_{\rm FBW, VSWR=2}$ (Sim)**	**$Q_{\rm exact}$ (Sim)**	**ΔQ_z *(%)***	***ka***
Edge	0.75	0	2.36	69.7	88.5	91.4	**81.3**	27%	2.3
Edge	0.75	31	0.54	162.0	108.1	157.1	**95.4**	33%	0.56
Edge	1.5	31	0.58	214.4	153.4	235.0	**126.6**	28%	0.58
Coax	0.75	0	2.33	46.1	52.1	55.1	**50.7**	13%	2.3
Coax	0.75	31	0.54	156.3	136.2	32.0	**113.8**	12%	0.56
Coax	1.5	31	0.58	210.4	200.3	47.5	**141.8**	4%	0.58

A major difference of microstrip from wire antennas is that the former have a parallel first resonance and thus a high $Z_{\rm in}$ at their edge. This difference is intensified for antennas that are electrically small. Hence, the edge-fed microstrips are highly mismatched. From the ${\rm Re}\{S_{11}\}$ and ${\rm Im}\{S_{11}\}$, the exact $Z_{\rm in}$ was extracted and used in (3) to obtain antenna Q.

The Q of the edge-fed rectangular antennas (with slit length $L_{\rm s} = 0$ mm) and meandered antennas (with $L_{\rm s} = 31$ mm) was calculated approximately from the measured and simulated $Z_{\rm in}$ using (3). The results were compared to the simulated exact Q_0 defined in (1) and the approximate $Q_{{\rm FBW}_V}$ from (4) with $s = 2$. The integrals in (1) were evaluated numerically for each observation angle and frequency to accurately compute the frequency derivative with the midpoint trapezoid rule, using the simulated radiation pattern data.

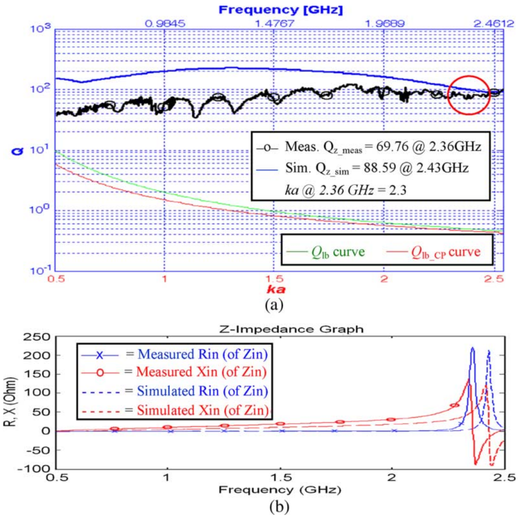
All simulated and measured results are presented in Table I.

A. *Rectangular, Edge-Fed Microstrip, $h = 0.75$ mm*

The Q and $Z_{\rm in}$ of the rectangular edge-fed microstrip antenna are shown in Fig. 3. The antenna is mismatched at all measured frequencies. The $Q_{\rm z}$ from (3) shows good agreement between simulations ($Q_{\rm z} = 88.5$) and measurements ($Q_{\rm z} = 69.7$) at the 2.36-GHz resonance. Although the measured $Z_{\rm in}$ appears smooth, minor fluctuations in ohmic losses and in its measurement, especially at the lower frequencies where $Z_{\rm in}$ is very small, are difficult to account and can result in the visible oscillation. The general behavior of the measured Q, however, follows the simulated trend. This $Q_{\rm z}$ is also compared to the $Q_{\rm lb}$ Chu limit for a lossless LP antenna. The circular polarization limit is shown for comparison.

B. *Rectangular, Coax-Fed Microstrip, $h = 0.75$ mm*

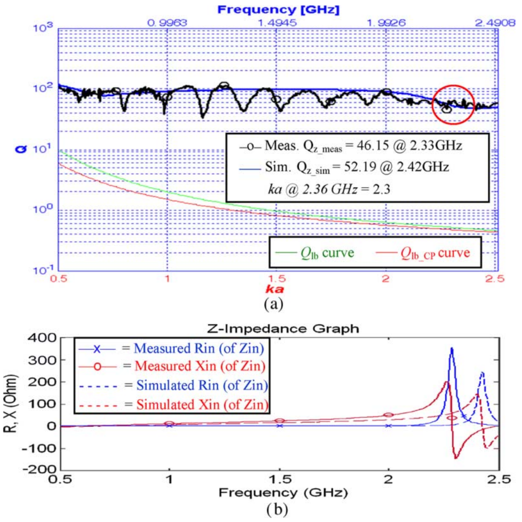
The rectangular coax-fed microstrip antenna Q and input impedance appear in Fig. 4. This antenna is well matched, and



(a)

(b)

Fig. 3. (a) Q and (b) input impedance of rectangular edge-fed microstrip antenna. Very good accuracy is observed near the resonance $f_r = 2.36$ GHz.



(a)

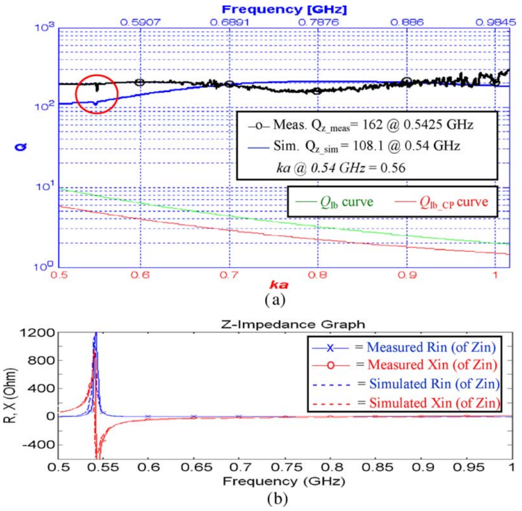
(b)

Fig. 4. (a) Q and (b) input impedance of rectangular coaxial-fed microstrip antenna. Good accuracy is observed near the resonance $f_r = 2.36$ GHz.

the measured Q agrees very well with the simulated, especially near the resonance $f_r = 2.36$ GHz. Here, the measured Q is very close to the simulated curve even at lower frequencies. This could be due to the improved feed matching and modeling, and the smaller VSWR on the feedline (less power reflected back to the source). All coaxial-fed antenna measurements were closer to the simulations than those of edge-fed antennas. Again, the measured Z_{in} does not fluctuate. The measured Q_z is 46.15 and is smaller than the edge-fed $Q_z = 69.7$. This provides a quantitative measure of how close the Q can be in practice for different feeding types. As a rule of thumb, a variation up to almost 30% was noticed between different feeds. In conjunction with the relatively large values of Q, this justifies the traditional plotting of Q in log-scale.

C. Meander, Edge-Fed Microstrip, $h = 0.75$ *mm*

The Q of miniaturized antennas is studied next. The meander antenna resonates at 0.54 GHz, so its resonant length is only $\lambda_0/15$ and height $\lambda_0/740$. The edge-fed antenna is mismatched at all measured frequencies 0.5–2.5 GHz. The measured Z_{in} is very smooth, and the resonance dip at $f_r = 0.54$ GHz is clearly observed on both curves in Fig. 5. The measured $Q_z = 162$ can be assumed that is relatively close to the simulated $Q_z = 108$.



(a)

(b)

Fig. 5. (a) Q and (b) input impedance of the meandered edge-fed microstrip antenna. The high input impedance at resonance can be noticed.

The electrically small edge-fed microstrip antennas have an input resistance in the vicinity of hundreds of ohm, which is much higher than that of traditional electrically small wire antennas that is typically less than 10 Ω. This happens because the first resonance of microstrips is of parallel nature, while that of wires is a series one. At resonance the microstrip antennas have real Z_{in} that is matched with a transformer or a coaxial feed that is placed at the correct point as shown next.

D. Meander, Coaxial-Fed Microstrip, $h = 0.75$ *mm*

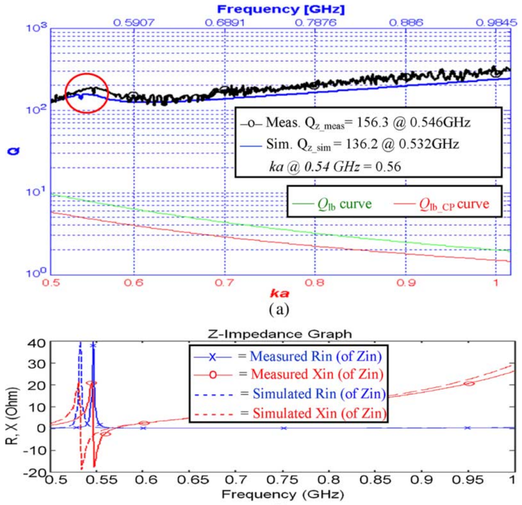
The first issue to resolve on coaxial-fed meandered miniaturized antennas is *how* to match them. Fig. 5(b) showed the edge-fed patch has $R_{\text{in}} \approx 1200\ \Omega$ at resonance. This is the highest value of Z_{in} and is measured at its edge where there is minimum input current and maximum input voltage. This value is notably higher than the 200-Ω R_{in} of regular patches. We know that, at the center of the patch, current is maximized and voltage is small, resulting in $Z_{\text{in}} \approx 0\ \Omega$. Since $Z_{\text{in}} \in (0\ \Omega, 1200\ \Omega)$, there must be a point on the structure where $Z_{\text{in}} = 50\ \Omega$. This point is close to the center of the surface and a simple via suffices to feed the antenna as in Fig. 2(d).

The Q is shown in Fig. 6, and excellent agreement between simulations and measurements is observed, especially near the resonance $f_r = 0.54$ GHz. This measurement is a solid experimental validation of (3), in that the Q can be found from Z_{in} irrespective of the type of feed. Here, the measured $Q_{\text{z\_meas}} = 156$ and the simulated $Q_{\text{z\_sim}} = 136$ are close to the previous edge-fed Q (162 and 108, respectively).

IV. ANTENNA Q FOR DIFFERENT SUBSTRATE THICKNESS

In the interest of a substrate thickness study, two more antennas were fabricated on thicker ($h = 1.5$ mm) substrate.

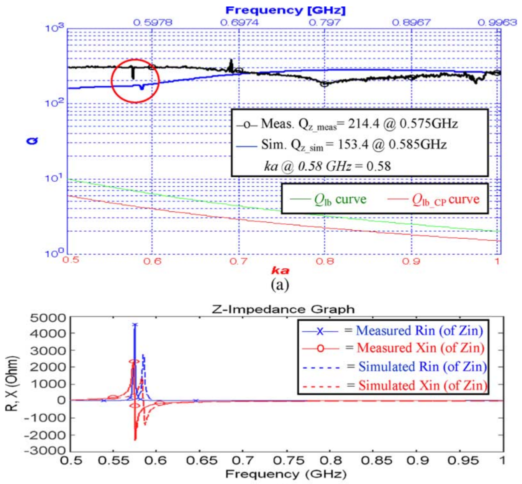
1) *Meander, Edge-Fed*, $h = 1.5$ mm, $L_s = 31$ mm*:* Resonates at 0.58 GHz and has resonant length $\lambda_0/15$ and height $\lambda_0/370$. It is mismatched and $Q_{\text{z}_{\text{meas/sim}}}$ 214 versus 153 (Fig. 7).
2) *Meander, Coax-Fed*, $h = 1.5$ mm, $L_s = 1.5$ mm*:* Matched at resonance with extremely close measurements and simulations. Its $Q_{\text{z\_meas/sim}}$ 210 versus 200, as the $h = 0.75$ mm one, and Q and Z_{in} are in Fig. 8.



(a)

(b)

Fig. 6. (a) Q and (b) input impedance of the meandered coaxial-fed microstrip antenna. The antenna has very similar measured and simulated Q.



(a)

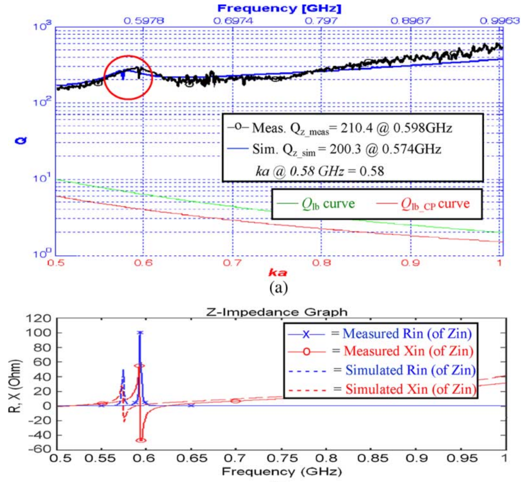
(b)

Fig. 7. (a) Q and (b) input impedance of the meandered edge-fed microstrip antenna with $h = 1.5$ mm showing measured results similar to the simulated.

V. Discussion and Conclusion

All slitted antennas are electrically small as they satisfy the Chu and Wheeler limit $k\alpha < 1$. Frequency reduction of up to 80% and metallization up to 44% are achieved. The Q of all models was calculated by approximate and exact formulas and the inverse bandwidth and Q relation. The finite dielectric caused a frequency shift that did not alter any outcome.

Obtained results showed that Q is prone to fluctuations of Z_{in} and requires accurate measurement, especially when Z_{in} is small. Measurements in general follow the simulated curves, and accuracy improves as Z_{in} increases. Coaxial-fed antennas provide notably more accurate Q curves, while a deviation of about 30% should be expected for edge-fed antennas, possibly due to improved port modeling and calibration (edge ports neglect the microstrip-coaxial junction), edge feed inductance, or high ohmic losses when Z_{in} is small. Although matching should not theoretically affect Q, it is shown that a good match enhances the measurement accuracy in practical cases. Also, despite the measured loss, the $Q_{\text{z\_meas}}$ may exceed the $Q_{\text{z\_sim}}$. A practical validation of (3) was made and showed reasonably similar measured and simulated Q curves even for mismatched antennas away from resonance. Results supported the fact that smaller antennas have narrower bandwidth (no magnetic materials used), and thus higher Q (meanders had 2× or more Q_{z} than rectangular microstrips). In all cases, Q is well above Q_{lb}, which indicates a potential for improvement. It is also of interest that a smooth measured Q curve represented a good calibration, while spikes indicated a faulty cable.



(a)

(b)

Fig. 8. (a) Q and (b) input impedance of the meandered coaxial-fed microstrip antenna with $h = 1.5$ mm showing measured results similar to the simulated.

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